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AUTHORS' CLOSURE

The authors are glad that their recent paper (Silva and Urgueira, 1988) presenting an analytical model for the out-of-plane vibration response of planar curved beams was considered of interest and raised some comments. In fact, it is not only gratifying but also very useful to have such an exchange of ideas in the large and heterogeneous community of researchers.

Following up the discussion, the authors would like to make it clear that the development of their analytical model was indeed based on the equations of equilibrium presented by Rao (1971) in his remarkable work. However, the authors cannot glimpse in their work any sign which may suggest that Rao's formulation suffers from any inaccuracy. The reason for not making a straightforward use of Rao's formulation is simply due to the fact that it is as concise as the number of response variables (the transverse displacement and the torsional angle) used in eqns (16) and (17) of Rao (1971). The objective of the authors was the development of an analytical model which could be easily used in the substructure approach. Therefore an additional equation was necessary in order to express the dependency of the out-of-plane flexural angle in terms of the corresponding transverse displacement [eqn (14) in Silva and Urgueira (1988)]. This approach made it possible to accomplish the final goal—to relate each dynamic excitation to each dynamic response in the out-of-plane harmonic motion of a curved element, and vice-versa, as is presented by Bishop and Johnson (1960) for the case of straight beams.

The second and not less important aspect is related to the effects of energy dissipation. The way chosen to tackle the problem is based on the workable approach used in the case of straight beams (Silva, 1978). Unfortunately, it is physically impossible to present any results now, but it is scheduled for the near future.

It is also suggested that the authors state the main differences between their method and Wang *et al.*'s (1984). To the authors' belief, the routes used to achieve the dynamic stiffness matrix have no significant differences. The major difference arises just at the beginning where the equilibrium equations, in the case of the paper presented by Wang *et al.* (1984), suffer from incompleteness in terms of the torsional inertia effects. One can say that these effects establish the main (but not the only) difference between the out-of-plane dynamic behaviour of a curved beam and the corresponding straight beam.

It is interesting to notice that the main disagreement on the natural frequencies predicted by both methods [Fig. 5 in Silva and Urgueira (1988)] is not only referred to very small curvatures. As can be seen in the graph of Fig. 6 in Silva and Urgueira (1988), which represents the transferred inertance predicted for a 60° curved beam, there are six elastic modes instead of four as predicted by Wang *et al.*'s model.

In fact, as the discussers have mentioned, the coupling between torsional and flexural vibrations tends to vanish as the configuration tends to a straight beam. It is therefore important to highlight the fact that, for small curvatures, the natural frequencies predicted by the authors' model indeed tend to those pertaining to a corresponding straight beam.

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REFERENCES

Bishop, R. E. D. and Johnson, D. C. (1960). *The Mechanics of Vibration*. Cambridge University Press, Cambridge.

Rao, S. S. (1971). Effects of transverse shear and rotary inertia on the coupled twist-bending vibrations of circular rings. *J. Sound Vibr.* **16**, 551–566.

Silva, J. M. M. (1978). Measurements and applications of structural mobility data for the vibration analysis of complex structures. Ph.D. thesis, Imperial College of Science and Technology, University of London.

Silva, J. M. M. and Urgueira, A. P. V. (1988). Out-of-plane dynamic response of curved beams—an analytical model. *Int. J. Solids Structures* **24**, 271–284.

Wang, T. M., Laskey, A. and Ahmad, M. (1984). Natural frequencies for out-of-plane vibrations of continuous curved beams considering shear and rotary inertia. *Int. J. Solids Structures* **20**, 257–265.